# GROWTH AND YIELD MODELS FOR PINUS RADIATA IN TASMANIA

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ABSTRACT

A set of growth and yield models for intensively managed **Pinus radiata** D. Don plantations in Tasmania have been constructed so that temporary inventory plot data can be used as the starting point for the simulation of stand dynamics. Stand-level increment models have been derived for mean dominant height, basal area, mortality, and volume. A combined parameter prediction and recovery method was used to forecast the parameters of the Weibull distribution which was used as a model of the diameter distribution. A model for the simulation of thinning has also been constructed.

Where possible the models have been cast as generalised linear models and a quasi-likelihood approach was adopted in the modelling of the random component of each model, resulting in an iteratively reweighted least squares procedure for the estimation of model parameters. A Poisson-like variance function was used to model the error variance of forecasts of stand variables - mean dominant height, basal area, and volume. Binomial or binomial-like variance functions were used for the mortality, diameter distribution, and thinning models.

**Keywords:** growth; yield; Weibull distribution; generalised linear models; **Pinus radiata.**

## INTRODUCTION

The Tasmanian Forestry Commission currently manages 38 000 ha of *Pinus radiata* plantations with an expected annual planting rate of 800 ha. This represents a relatively small area compared to the area of native forests managed but the *P. radiata* resource is a relatively high-value crop. This is especially true for pruned stands from which a large volume of wood will be available from about 1990 onward. Prior to 1973 stands were mostly planted at either 1.83 × 1.83 m or 2.44 × 2.44 m spacing, the latter being the more prevalent, and were usually treated with frequent, late, and light thinnings. After 1973 intensive management regimes were adopted (Neilsen & Davis 1985). These regimes were based on New Zealand research (Fenton & Sutton 1968) and characterised by few, early, and heavy thinnings. Trees were generally planted at a spacing of 5 × 2.5 m for stands to be pruned (clearwood regime) and 3 × 2.5 m for non-clearwood stands.

A simulation model which predicts volumes and piece sizes using the current temporary plot inventory system is required for long-term resource planning, and for 5-yearly and yearly operational planning. This paper describes models of mean dominant height (MDH) increment (from which a family of anamorphic site index curves can be obtained), basal area increment, volume (total volume from ground to tip) increment, mortality, and diameter (DBH) distribution, and one which allows the simulation of a thinning. The basal area increment function uses a stand density index developed by Lawrence (1976).

The temporary plot inventory system provides the starting point for the simulator. The various stand variables are calculated from these plots at the measurement age and an increment function is applied for MDH, basal area, volume, and mortality (or a decrement function for stocking). In the DBH distribution model a hybrid of what Hyink & Moser (1983) called parameter prediction and parameter recovery methods is used to predict the parameters of the Weibull probability density function (*pdf*) which is assumed to describe the DBH distribution. A parameter prediction method is used to obtain the model which distributes the thinnings across the DBH classes.

This paper has a dual purpose. Firstly, it describes the above models, their form, application, and, for volume, performance. Secondly, it describes the statistical methodology used to develop the models. This methodology involves generalised linear models and quasi-likelihood theory which are relatively recent developments in statistics. Some readers will prefer to ignore some of the details of the statistical methods and this can be done without loss of understanding of the models and their application.

## Notation

| | |
|---|---|
| $k$ | "measurement" number |
| $y_k$ | stand variable at measurement $k$ |
| $t_k$ | age at measurement $k$ |
| $\mu_k$ | expected value (mean) of $y_k$ conditional on the model |
| V(.) | variance function |
| $\Phi$, $\lambda$ | dispersion and variance function parameters |
| $\alpha$, $\beta$ | parameters for increment models, Weibull shape parameter model and thinning model |
| $c$, $\theta$, $d_0$ | Weibull *pdf* shape, scale, and location parameters |
| $A$ | asymptote parameter |
| $D$ | DBH over bark (cm) |
| $H$ | mean dominant height (m) |
| $S$ | site index (m) (mean dominant height at age 20) |
| $V$ | stand volume ($m^3$/ha) |
| $B$ | basal area ($m^2$/ha) |
| $B_t$ | maximum basal area ($m^2$/ha) |
| $S_d$ | stand density index (%) |
| $S_t$ | reduction in $S_d$ due to thinning (%) |
| $P_r$ | pruned ratio (pruned height/$H$) |
| $N$ | stocking (stems/ha) |
| $N^*$ | number of stems on the plot |
| $M_k$ | mortality between ages $t_k$ and $b_{k-1}$ (stems/ha) |
| $K$ | $\pi/[4(100^2)]$ constant to convert $DBH^2$ to basal area |
| $Q$ | $\sqrt{\{B/(K\ N)\}}$ quadratic mean DBH (cm) |
| $T$ | years since last thinning or time between two ages |

## DATA

Three series of research plots were used to construct the models. They were:

*Plantation Yield Plots (PYP):* These plots consist of a large number of permanent 0.06-ha rectangular plots established in the old regimes and measured generally every 3 years. The data from unthinned *PYP* were used to develop the stand density index and the mortality function.

*Plantation Growth Plots (PGP):* These plots are the current series of permanent plots with plots established after 1973, in general, treated under the new regimes. These plots are rectangular and generally 0.08 ha in size. About 56% of the current number of 405 plots were established as *PYP* and converted to *PGP* by treating them (i.e., thinning and/or pruning) in line with the new regimes.

*Thinning plots:* Five series of these permanent plots have been established. Each series consists of 20, 0.06-ha, contiguous plots. For each plot a different thinning and/or pruning regime is applied. These plots, as with the *PGP*, are measured annually but differ from the *PGP* in their layout and the fact that the treatments cover a wider range of thinnings than currently used in practice (and sampled by the *PGP* series). For the following, *PGP* will refer to both the *PGP* and thinning series of plots.

Thinnings were removed in a way that allowed retention of good form, larger stems with form taking precedence over size in selection for retention. Some plots were row thinned with extra thinnings obtained from between the outrows and retained trees selected as above.

Pruning involved three lifts of 2.1, 4.3, and 6.4 m.

Since the models to be constructed were stand-level models, stand-level data were obtained at the first measurement and then at each measurement at which a thinning or pruning occurred and finally at the last measurement. Measurements at which no treatment of the stand occurred were ignored. This was done to make the database more manageable and reduce the influence that multiple measurements would have on statistical tests of model parameters (West *et al.* 1984). The resultant loss of information was limited by the high degree of serial correlation between consecutive measurements. Information on the shape of the basal area growth response to thinning and pruning is sacrificed at an individual-plot level by doing this. However, such growth responses can still be estimated from the data across plots. The full data set would be required to carry out a two-stage or random coefficients approach (West *et al.* 1984) to modelling but such an approach is impractical with models as complex as the basal area increment function developed here.

As a result, 847 measurements on 359 plots were available for model building where, for the following, references to measurements will be taken to mean this reduced data set. For basal area and volume, although the at-thinning measurement of the *PGP* plots is a single measurement, these stand variables are calculated both before and after thinning. This ambiguity is avoided by the convention, used for the following, that for both the fit of the models and their application (a) if basal area or volume, at thinning, are forecast values (or observed values to be compared to a forecast) they refer to standing plus felled but excluding dead trees, and (b) if they are initial values at the start of a forecast period they refer only to standing live trees. All basal area and volume figures are net values which exclude mortality. A summary of the *PGP* plot data is given in Table 1. Volume data were derived using the individual-tree volume equation described by Candy (1989) which required as input the tree DBH, total height, age, and bark thickness. To obtain tree total heights a sample of between 15 and 20 trees were measured on *PGP* plots, with selection of sample trees weighted towards the larger DBH classes. The total height of the unmeasured trees was obtained using a regression of the logarithm of total height on the reciprocal of DBH applied to the plot measurement using the above sample.

TABLE 1–*PGP* plot data summary—Stand variables

| Stand variable | Mean | Minimum | Maximum |
|---|---|---|---|
| Age (years) | 12.8 | 4.0 | 40.0 |
| Site index (m) (MDH at age 20) | 27.8 | 16.4 | 42.0 |
| MDH (m) | 18.5 | 5.0 | 43.8 |
| Stocking (stems/ha) | 689.1 | 166.7 | 1866.0 |
| Basal area ($m^2$/ha) | 27.9 | 1.5 | 109.1 |
| Number of "measurements" | 1.9 | 1.0 | 4.0 |
| Time interval between "measurements" (years) | 3.9 | 0.2 | 19.0 |
| Number of thinnings | 1.6 | 0.0 | 3.0 |
| Pruning height (%) | 15.5 | 12.1 | 70.5 |
| Stocking removed (%) | 46.7 | 9.7 | 85.9 |
| Basal area removed (%) | 38.9 | 6.2 | 78.4 |
| Maximum basal area ($m^2$/ha) (function of MDH) | 49.4 | 3.3 | 98.4 |
| Stand density index (%) | 47.7 | 8.2 | 150.0 |
| Mean mortality rate before thinning (stems/ha/year) | 4.6 | 0.0 | 46.5 |
| Mean mortality rate after thinning (stems/ha/year) | 3.6 | 0.0 | 73.7 |
| Volume ($m^3$/ha) | 181.2 | 3.0 | 1256.0 |
| CAI ($m^3$/ha/year) | 23.8 | 1.9 | 78.1 |

For the DBH distribution model, a DBH class frequency table using 4-cm classes from 6 cm onwards and a top class of 50 cm and greater was constructed for each measurement. This gave 12 classes. To construct the thinning model the DBH class frequency table was constructed before and after thinning for each at-thinning measurement but with trees removed by row thinning excluded. A total of 548 thinning measurements of 303 plots was obtained in this way.

The mortality model was constructed from 1282 measurements of 285 unthinned *PYP* plots. Of the 1282 measurements, 1198 were taken on plots in stands of nominal square spacing of either 1.83 or 2.44 m (Table 2).

TABLE 2–*PYP* plot data summary. Summary of the data for 1282 measurements of the unthinned *PYP* plots used to construct the mortality function as described in the text.

| Stand variable | Mean | Minimum | Maximum |
|---|---|---|---|
| Age (years) | 13.1 | 6.1 | 44.0 |
| Site index (m) (MDH at age 20) | 29.3 | 19.9 | 39.0 |
| Stocking (stems/ha) | 2028.0 | 650.0 | 3983.0 |
| Number of measurements | 4.4 | 4.0 | 6.0 |
| Time interval between measurements (years) | 4.5 | 0.3 | 30.3 |
| Mean mortality rate (stems/ha/year) | 27.8 | 0.0 | 425.9 |

## STATISTICAL METHODOLOGY

The general form of the increment model used here for a general stand variable, $y$, is given by

$$y_k = y_{k-1} f(Y_{k-1}, t_k, t_{k-1}, \beta) + \varepsilon_k \tag{1}$$

where

$y_k$, $y_{k-1}$ are the values of $y$ at ages $t_k, t_{k-1}$ $(t_{k-1} < t_k)$
$Y_{k-1}$ is a vector of stand variables at age $t_{k-1}$
$\beta$ is a vector of unknown parameters
$\varepsilon_k$ is a random error.

Thus the value of a stand variable, $y$, at time $t_k$ is forecast conditional on its own value and that of other stand variables at time $t_{k-1}$. Although for the following reference will be made to increment models, the form of the model will be given by Eqn (1). The increment, $\Delta y_k = y_k - y_{k-1}$, is implied in Eqn (1) and is given by

$$\Delta y_k = y_{k-1}[f(Y_{k-1}, t_k, t_{k-1}, \beta) - 1] + \varepsilon_k.$$

Note that the error term, $\varepsilon$, and thus the residual from the fitted model, is the same whether the model is expressed in terms of $y_k$ or $\Delta y_k$. The mortality and volume increment functions are slight variations of this general form.

Given the high positive (serial) correlation of the values of a stand variable over time between stand treatments then, from a known point (i.e., age $t_{k-1}$), the observed and forecast trajectories will tend to diverge. Therefore, for the MDH, basal area, and volume increment models a variance function of the following form was used where

$$Var(\varepsilon_k) = \Phi \, V(\mu_k) \tag{2}$$
$$V(\mu_k) = (\mu_k - y_{k-1})^\lambda$$
$$\mu_k = E(y_k \mid Y_{k-1}, t_k, t_{k-1}, \beta) \text{ and } \Phi \text{ is a dispersion parameter.}$$

The variance function is given by V(.) and $\mu_k$ is the conditional expectation of $y_k$. Attention was restricted to two particular functions, the gamma-like ($\lambda$=2) and the Poisson-like ($\lambda$=1). The resulting variance functions are analogues of the variance functions for the gamma and Poisson distributions respectively. These functions allow the variance of $y_k$ about its forecast, $\hat{\mu}_k$, to increase as the forecast increment $(\hat{\mu}_k - y_{k-1})$ increases, which mimics the divergence of trajectories described above.

Thus, no distributional assumptions are made concerning $y$ but rather the assumption specified by Eqn (2) is used to determine estimation and testing procedures based on the quasi-likelihood theory developed by Wedderburn (1974). In practical terms the estimation of $\beta$ is carried out by an iteratively reweighted least squares (*IRLS*) algorithm with iterative weights given by $1/V(\hat{\mu}_k)$. An estimate of $\Phi$ can be obtained as the residual mean deviance (McCullagh & Nelder 1983). The deviance is the likelihood (or quasi-likelihood in this case) equivalent to the residual sum of squares for normal distribution (or constant error variance, $\lambda=0$) models and is used in the same way to test the significance of the change in the deviance due to fitting a series of nested models. The mean residual deviance is the equivalent to the residual mean square. The *IRLS* used-defined-model facility availble in *GLIM* (Numerical Algorithms Group 1985) was used to fit the MDH, basal area, volume increment, and mortality models where the variance function in this last case is of a slightly different form to Eqn (2) as described below. The deviance functions corresponding to $\lambda=1,2$ are derived in the Appendix.

For the mortality model, a binomial-like variance function which has a similar effect to the Poisson-like function described above was used. If $y_k$ is stocking ($N$) at age $t_k$ and mortality is given by $M_k = y_{k-1}-y_k$ then this variance function, conditional on $y_{k-1}$, is given by

$$V(\mu_k) = \mu_k(y_{k-1}-\mu_k) \tag{3}$$

where

$$\mu_k = E(M_k \mid Y_{k-1},t_k,t_{k-1},\beta).$$

The above variance function is equivalent to that obtained by scaling the binomial variance function, obtained by considering mortality as binomially distributed conditional on $y_{k-1}$, by multiplying by $y_{k-1}$. If the binomial variance function is used then the assumption that the variance of the proportion mortality (i.e., $M_k/y_{k-1}$) decreases as the binomial sample size $y^*_{k-1}$ ( $= N^*$, the number of trees on the plot), increases is implied. The above variance function can be used to model the conditional variance of any stand variable that is constrained to lie between zero and a known value (i.e., $y_{k-1}$ in this case). The corresponding deviance function for Eqn (3) is not derived here since it is simply that for the binomial (*see* Nelder & Wedderburn 1972) divided by $y_{k-1}$ and can be employed in either *GLIM* or *GENSTAT* (Numerical Algorithms Group 1983) using a binomial distribution with prior weights given by $1/y_{k-1}$.

The applicability of a particular variance function was tested by plotting generalised Pearson residuals (McCullagh & Nelder 1983) given by $(y_k-\hat{\mu}_k)/\sqrt{V(\hat{\mu}_k)}$ against the fitted values, $\hat{\mu}_k$ or $\hat{\mu}_k-y_{k-1}$ in the case of increment models. The Pearson residuals should appear more homoscedastic than standard normal residuals ($\lambda=0$), $y_k-\hat{\mu}_k$, if the variance function models reasonably well the conditional variance of $y$.

## MDH Increment/Site Index Model

A form of the widely used Richards (1959) model was used to model the MDH/age relationship. This model is defined as

$$H = A(1-exp\{-\alpha t\})^{\beta} \tag{4}$$

where $H$ is MDH, $t$ is age, and $A$, $\alpha$, $\beta$ are parameters, $A$ being the asymptote. This model can be expressed conditional on known $H$ at age $t_{k-1}$ (i.e., $H_{k-1}$), where $k$ indexes one in a series of ages, as

$$H_k = H_{k-1}[(1-exp\ \{ -\alpha\ t_k \}\ )/1-exp\ \{ -\alpha\ t_{k-1} \}\ )]^{\beta} \tag{5}$$

This model was fitted to the data using the MDH/age series from the *PGP* plots with $H_{i\,1},\ldots,H_{i\,,n_i-1}/t_{i\,1},\ldots,t_{i\,,n_i-1}$ as independent variables in the fit with $H_{i2},\ldots H_{i\,,n_i}$ as corresponding dependent variables where the $i$ subscript is introduced to refer to plot and $n_i$ is the number of measurements on the $i$th plot.

If $t_{k-1}$ is replaced by 20, $t_k$ by the general term $t$, $H_{k-1}$ by site index, $S$, and $H_k$ by the general term $H$ in (5), then the result

$$H = S[(1-exp\{ -\alpha\ t \})/(1-exp\{ -\alpha\ 20 \}\ )]^{\beta} \tag{6}$$

defines a family of anamorphic site index curves with $A$ being the indexing parameter. Given a known MDH (i.e., from a measurement of a temporary plot), then site index can be estimated by solving Eqn (6) for $S$. An alternative polymorphic series using the Richards model was used by Garcia (1983) by defining the shape parameter $\alpha$ as the indexing parameter. This assumes that the MDH converge to a single asymptote independent of site index. Such a trend was not observed in the data but rather the anamorphic series indexed by $A$ was indicated, and so this form of the site index model will be retained for the following.

The conditional model was the form of the MDH/age relationship fitted here although a different parametrisation was used. The generalised linear model, *glm*, (Nelder & Wedderburn 1972) parametrisation was used to allow the *IRLS* algorithm available in *GLIM* to use variance functions of the class described by Eqn (2). The *glm* parametrisation of Eqn (5), using Thompson & Baker's (1981) composite link functions, is given by

$$H_k = H_{k-1}exp\{ \beta[ln(1-exp\{ -\alpha\ t_k \})-ln(1-exp\{ -\alpha\ t_{k-1} \})] \} \tag{7}$$

Besides the availability of *IRLS* in *GLM*, the *glm* parametrisation, in the author's experience usually has few problems with lack of convergence of parameter estimates. The model given by Eqn (7) was initially fitted to the *PGP* data using unweighted least squares. Graphical examination of Pearson residuals for $\lambda$=0,1,2 *versus* predicted increment, $\hat{H}_k-H_{k-1}$, was carried out. The case of $\lambda$=1 was chosen for fitting the model using *IRLS*. This was also the choice for the basal area and volume increment models. Discussion of the relative merits of these variance functions for all three increment models is left to the end of the paper. The resulting estimates and their standard errors are given in Table 3. The resultant family of MDH/age curves for selected site indices is shown in Fig. 1.

**TABLE 3–Parameter estimates for mean dominant height (MDH) (Eqn 7)**
$H_k = H_{k-1}exp\{\beta[ln(1-exp\ \{-\alpha t_k\})-ln(1-exp\ \{-\alpha t_{k-1}\})]\}$

| Parameter | Estimate | Standard error |
|---|---|---|
| $\alpha$ | 0.06248 | 0.00176 |
| $\beta$ | 1.633 | 0.026 |
| | $\hat{\Phi}$ = 0.2769 | |

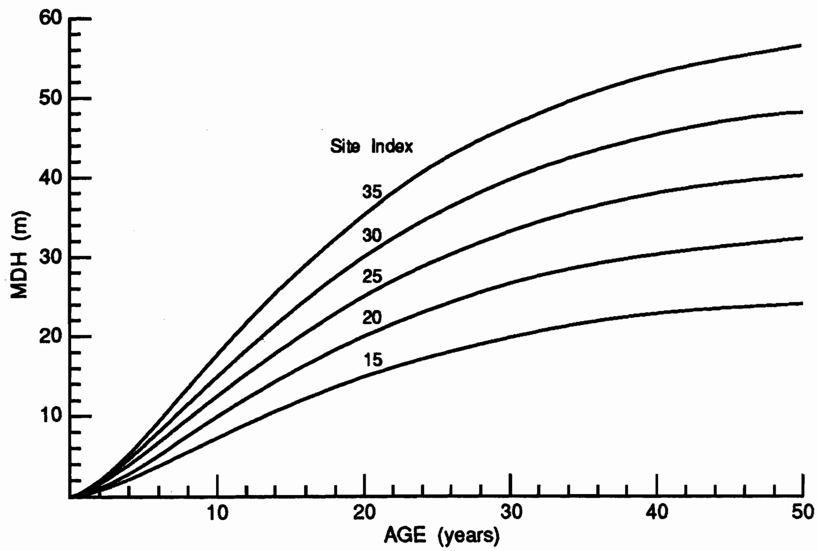


FIG. 1—Anamorphic site index curves.

## Basal Area Increment Model

Two forms of the basal area increment function were fitted. The first was the Richards model as given by Eqn (4) with $H$ replaced by basal area $B$. An alternative form based on a Gompertz function was also tested where the yield form of the model is given by

$$B = Aexp\{-exp[\alpha+\beta ln(t)]\} \qquad (8)$$

while the conditional form of the model is given in the *glm* parametrisation by

$$B_k = B_{k-1}exp\{exp[\alpha+\beta ln(t_{k-1})] - exp[+\beta ln(t_k)]\} \qquad (9)$$

The only difference between this model and the Gompertz is that in the Gompertz $ln(t)$ is replaced by $t$ in the above equations. The author has found the model based on Eqn (8) as good as, if not superior to other three-parameter asymptotic yield functions in its ability to model different shapes of yield/age relationships.

In both the Richards- and Gompertz-like models the parameters $\alpha$ and $\beta$ were replaced by models incorporating combinations of the stand variables: site index ($S$), stand density index ($S_d$), thinning index ($S_t$) (i.e., the reduction in $S_d$ due to thinning), time since last thinning ($T$), and pruned height ratio ($P_r$ ).The stand density index used was constructed by Lawrence (1976) using 2120 measurements of 790 unthinned *PYP*. The index is given by 100 $B/B_f$ where $B_f$ is the predicted maximum basal area for a stand of given MDH (i.e., a fully stocked stand) and $B_f$ is given by

$$B_f = 229.568\ ln\{1.7399 - 0.8633/exp(H/30.48)\}.$$

This model was constructed by grouping the measurements in 1.5-m MDH classes and calculating the mean of the top 5% of stand basal areas. The model was obtained by nonlinear regression using the above mean basal areas as the dependent variable. Alder (1979) developed a similar relationship between maximum stand basal area and dominant height using a Richards model. Lawrence found that $B_f$ given MDH was independent of site index. Thus stand density index can be quoted as the percentage of maximum basal area that current basal area represents. The trajectory of the $B_f$ *versus* age curve for selected site indices is shown in Fig. 2. A referee has pointed out that before regular (i.e., density-dependent) mortality commences the definition of maximum basal area is heavily dependent on the range of stockings in the *PYP* database. This is a valid point but in application only initial stockings greater than roughly 3000 stems/ha would be outside the range of the *PYP* data and such high initial stockings are not used in Tasmania. The thinning index used was simply 100 $B_t/B_f$ where $B_t$ is the basal area removed in thinning. Time since last thinning ($T_k$) is the number of years to age $t_k$ from the last thinning. The pruned ratio is the ratio of pruned height to MDH at the time of pruning and is zero for measurements other than at pruning.

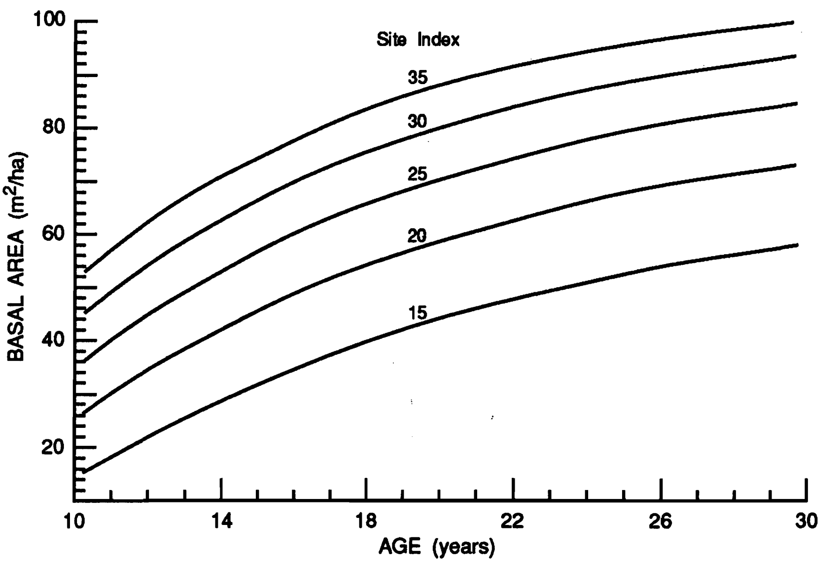


FIG. 2—Maximum basal area by site index.

Of all the models tested the best model in terms of residual deviance, based on $\lambda=1$, was that based on Eqn (9) but with $\alpha$ varying with $k$ where

$$\alpha=\alpha_k=\alpha_0+\alpha_1 S+\alpha_2 S_d+\alpha_3 S_d^2+\alpha_4 S_t+\alpha_5 S_t T_k+\alpha_6 P_r \qquad (10)$$

$$\beta=\beta_0+\beta_1 S+\beta_2 S_d+\beta_3 S_d^2+\beta_4 P_r.$$

The resulting parameter estimates and their standard errors are given in Table 4. The addition of a location parameter to the model by in effect replacing $t$ with $t$–$t_0$ in Eqn (8) did not significantly ($p > 0.10$) improve the fit of the model.

TABLE 4–Parameter estimates for basal area (Eqn 9, 10)

$$B_k = B_{k-1} exp\ \{exp[\alpha+\beta ln(t_{k-1})]-exp[\alpha+\beta ln(t_k)]\}$$
$$\alpha = \alpha_k = \alpha_0 + \alpha_1 S + \alpha_2 S_d + \alpha_3 S_d^{\,2} + \alpha_4 S_t + \alpha_5 S_t T_k + \alpha_6 P_r$$
$$\beta = \beta_0 + \beta_1 S + \beta_2 S_d + \beta_3 S_d^{\,2} + \beta_4 P_r$$

| Parameter | Estimate | Standard error |
|---|---|---|
| $\alpha_0$ | 3.396 | 0.125 |
| $\alpha_1$ | –0.02317 | 0.00304 |
| $\alpha_2$ | –0.01040 | 0.00293 |
| $\alpha_3$ | $1.381 \times 10^{-4}$ | $0.252 \times 10^{-4}$ |
| $\alpha_4$ | $0.3452 \times 10^{-2}$ | $0.0740 \times 10^{-2}$ |
| $\alpha_5$ | $-0.7331 \times 10^{-4}$ | $0.3627 \times 10^{-4}$ |
| $\alpha_6$ | –0.1338 | 0.0655 |
| $\beta_0$ | 0.2061 | 0.0661 |
| $\beta_1$ | –0.01457 | 0.00249 |
| $\beta_2$ | $-0.7553 \times 10^{-2}$ | $0.1404 \times 10^{-2}$ |
| $\beta_3$ | $-0.2655 \times 10^{-4}$ | $0.1326 \times 10^{-4}$ |
| $\beta_4$ | –0.1157 | 0.0521 |
| | $\hat{\Phi} = 0.6180$ | |

The base form of the Gompertz-like model given by Eqn (8) and (9) has the property of compatibility described by Clutter (1963) as a desirable property for any growth and yield system of models. By compatibility it is meant that the forecast of $B_{k+1}$ made conditional on $B_{k-1}$ and thus made in a single step is identical to the forecast of $B_{k+1}$ made in two steps the first of which forecasts $B_k$ conditional on $B_{k-1}$ and the second forecasts $B_{k+1}$ conditional on this forecast of $B_k$. However, the model given by incorporating the above models of $\alpha$ and $\beta$ does not have this property. The issue of compatibility does not arise in the fit of the model to the data because the model in this case always forecasts $B_{k+1}$ conditional on the known value of $B_k$ rather than a forecast value of $B_k$. In application of the model only one known initial value of basal area is available, the value at the measurement of the temporary plot. Therefore, to be consistent with the way the model was fitted, any future forecasts should be made conditional on the known values of $B$ and other stand variables up until the next simulated treatment of the stand where thinning and/or pruning is applied. The values of the stand variables are then recalculated at the treatment age and resultant values (after removing simulated thinnings) used as the initial starting point for further forecasts.

The property of compatibility could be restored by dropping the terms involving $S_d$ in Eqn (10) and, in application to forecasts between stand treatments, by carrying forward the values of $P_r$ and $S_t$ through to each forecast age. However, when terms involving $S_d$ were dropped from Eqn (10) and the model refitted there was a 53% increase in the residual deviance compared to the full (non-compatible) model. Given that the non-compatible model is applied in the same way that it was fitted to the data, as described above, then this model is recommended because of its greater precision of forecasts.

## Mortality Model

The mortality model was fitted to the *PYP* data for which stocking was sufficiently high and plots remained unthinned to sufficient ages for regular mortality to be adequately expressed. The general form of the model used was

$$M_k = N_{k-1} \left[1 - exp\left\{-\int_{t_{k-1}}^{t_k} \eta(t)dt\right\}\right] \tag{11}$$

where $M$ and $N$ are mortality and stocking respectively, both expressed as stems per hectare, and $\eta(t)$ is a function of stand variables as well as age. The best model was found to be $ln(\eta)$ a quadratic function of $t$ plus a linear component involving site index. To fit this model using *GLIM* the following approximation to the integral in Eqn (11) was used

$$\int_{t_{k-1}}^{t_k} \eta(t)\, dt \cong T\, exp(\beta_0 + \beta_1 S + \beta_2 T_m + \beta_3 T_m^2)$$

where $T = t_k - t_{k-1}$ and $T_m = (t_k + t_{k-1})/2$.

Examination of Pearson residuals suggested the variance function given by Eqn (3) was most appropriate. As a result the model given by Eqn (11) was fitted to the data using *IRLS* and *GLIM* using $1/N_{k-1}$ as prior weights and an assumed binomial distribution for $M_k$ conditional on $N_{k-1}$. The parameter estimates and their standard errors are given in Table 5. The stocking trajectory for selected initial stockings at age 10 combined with selected site indices is shown in Fig. 3.

TABLE 5–Parameter estimates for mortality (Eqn 11)

$$M_k = N_{k-1}\,[1 - exp\,\{-\int_{t_{k-1}}^{t_k} \eta(t)dt\}]$$

| Parameter | Estimate | Standard error |
|---|---|---|
| $\beta_0$ | −12.68 | 0.42 |
| $\beta_1$ | 0.1255 | 0.0123 |
| $\beta_2$ | 0.4458 | 0.0178 |
| $\beta_3$ | −0.00837 | 0.00045 |

The above class of model and estimation procedure is described by Candy (1986) for fitting survival models with a log-linear hazard (i.e., age-specific mortality rate) function (McCullagh & Nelder 1983). The only difference here is that the variance function (3) is used rather than a binomial. The above model gave a better fit to the data than the model based on the Weibull survival-time distribution, for which $ln(\eta)$ is a linear function of $ln(t)$, since its form of the hazard function is more flexible than that of the Weibull. The improved fit is noticeable when stands are sufficiently developed that the mortality rate slows down after most of the suppressed trees have died. For a general discussion of such effects of heterogeneity in populations see Vaupel & Yashin (1985).

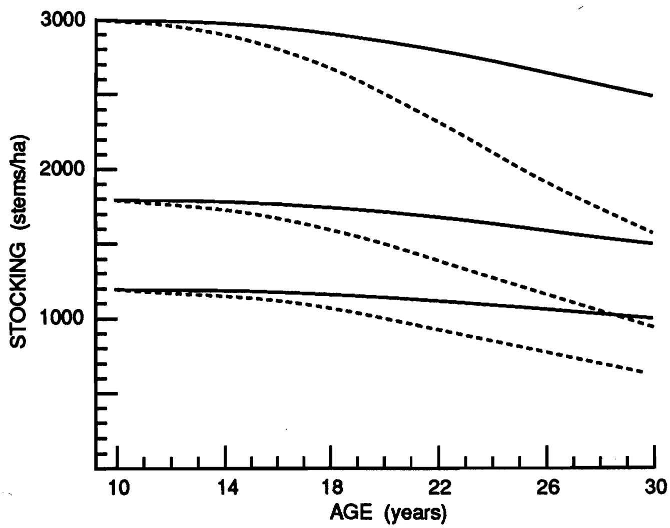


FIG. 3—Stocking curves for unthinned stands. Site index 30 (. . . . . .), 20 (————).

### Volume Increment Function

By defining the following relationship between volume $V$, and forecast values of $B$ and $H$

$$V = exp\{ \beta_0 + \beta_1 ln(H) + \beta_2 ln(B) \} \quad (12)$$

then the following conditional model can be derived

$$V_k = exp\{ lnV_{k-1} + \beta_1 ln(H_k/H_{k-1}) + \beta_2 ln(B_k/B_{k-1}) \} \quad (13)$$

where if measurement $k$–1 is taken at thinning then $V_{k-1}$ and $B_{k-1}$ are the volume and basal area, respectively, after removing thinnings. Both models (12) and (13) were fitted to the data using *IRLS*. The Poisson-like variance function was used to fit Eqn (13) and a gamma variance function for Eqn (12) (i.e., variance function the square of predicted volume not predicted increment). To compare the fit of these models directly the forecast values of $V_k$ from both models were used to calculate $\hat{\phi}$ using the deviance function based on the Poisson-like variance function. It was found that for thinned stands Eqn (13) was more accurate ($\hat{\phi} = 4.382$) than Eqn (12) ($\hat{\phi} = 4.756$). The accuracy for both models was considerably worse for unthinned stands but Eqn (12) was more accurate ($\hat{\phi} = 6.484$ compared to 8.949).

Eqn (12) is a variant of the well known Schumacher's yield model. Borders & Bailey (1986) used equivalent forms of Eqn (12) and Eqn (13). However, they used a predicted value for $V_{k-1}$ in Eqn (13) by applying Eqn (12) with $B = B_{k-1}$ and $H = H_{k-1}$. Here Eqn (13) simply reduces to Eqn (12). Using an actual value of $V_{k-1}$ (i.e., at the measurement of the *PGP* or, in application, the temporary inventory plot) is equivalent to allowing $\beta_0$ in Eqn (12) to vary as

$$\beta_0^* = ln\{ V_{k-1}/(H_{k-1}^{\ \beta 1}\ B_{k-1}^{\ \beta 2}) \} .$$

Thus if the timing, intensity or type of thinning affects $\beta_0^*$ then Eqn (13) takes this into account without an explicit model for the effect of thinning. For example, it was found that $\beta_0$* calculated using Eqn (13) estimates of $\beta_1$ and $\beta_2$ was significantly ($p < 0.001$) positively correlated ($R = 0.212$) with $S_t$ for thinned measurements.

In application, for forecasts from an age where a simulated thinning has been applied, $V_{k-1}$ is predicted rather than known; however, as described later in the section on application of the models, $V_{k-1}$ is predicted in a way that takes thinning into account.

The poorer accuracy of Eqn (13) for unthinned stands cannot be explained at this time so it is recommended that Eqn (12) be used for unthinned and Eqn (13) for thinned stands.

The parameter estimates for each model and $\hat{\Phi}$ are given in Table 6.

TABLE 6–Parameter estimates for the volume increment model

| | Parameter | Estimate | Standard error |
|---|---|---|---|
| Eqn 12 $V = exp\ \{\beta_0 + \beta_1 ln(H) + \beta_2 ln(B)\}$ | | | |
| | $\beta_0$ | –0.7466 | 0.0289 |
| | $\beta_1$ | 0.8579 | 0.0157 |
| | $\beta_2$ | 1.007 | 0.009 |
| | | $\hat{\Phi} = 0.01486$ (gamma) | |
| | | $= 5.088$ (Poisson-like) | |
| Eqn 13 $V_k = exp\ \{lnV_{k-1} + \beta_1 ln(H_k/H_{k-1}) + \beta_2 ln(B_k/B_{k-1})\}$ | | | |
| | $\beta_1$ | 0.8992 | 0.0254 |
| | $\beta_2$ | 1.001 | 0.025 |
| | | $\hat{\Phi} = 5.261$ | |

## Validation of the Volume Increment Function

Due to limitations on space it is not possible to present detailed results for each increment model here. The overall accuracy for each model is given in Tables 3 and 4 in the form of the estimate of $\Phi$. Greater detail is given here for the volume increment function since it is of most interest to users and its accuracy depends on the accuracy of forecasts from the basal area and MDH increment functions. Examination of residuals revealed no serious problems with these models.

The mean bias of forecasts of $V$, the standard error of the bias, mean predicted increment, and $\hat{\Phi}$ are given for the *PGP* data, using Eqn (13), for each cell of the site index class by age class table given in Table 7. The results are for thinned stands only. Note that the estimate of $\Phi$ is a function of both the precision and bias of forecasts.

An estimate of $\hat{\Phi}$, calculated as the mean residual deviance, can be used to estimate the prediction error variance of a single forecast of $V$. Based on Eqn (2) the variance of the prediction error (*PEV*) is given by

$$PEV(\hat{V}_k) = E\ \{\ (\hat{V}_k - V_k)^2\ \} \cong \hat{\Phi}(\hat{V}_k - V_{k-1}).$$

TABLE 7–Results for the volume increment function ($m^3$/ha, except for $\Phi$ and $n$). Results for cells where the number of plot-measurements, $n$, is less than 6 are not shown.

| Site index class | | Age class (years) <10 | 10–15 | 15–20 | 20–25 | 25–30 | >30 |
|---|---|---|---|---|---|---|---|
| <20 | Mean bias | –1.3 | –2.1 | – | – | – | – |
| | s.e. (bias) | 0.8 | 0.8 | – | – | – | – |
| | Pred. inc. | 11.5 | 11.3 | – | – | – | – |
| | $\Phi$ | 0.47 | 0.60 | – | – | – | – |
| | $n$ | 6 | 9 | – | – | – | – |
| 20–25 | Mean bias | –0.4 | –1.3 | 10.7 | – | – | – |
| | s.e. (bias) | 1.0 | 0.8 | 3.6 | – | – | – |
| | Pred. inc. | 17.1 | 17.3 | 72.0 | – | – | – |
| | $\Phi$ | 1.15 | 0.64 | 4.32 | – | – | – |
| | $n$ | 30 | 43 | 25 | – | – | – |
| 25–30 | Mean bias | –0.3 | –4.0 | 4.9 | –3.6 | –11.7 | –10.5 |
| | s.e. (bias) | 0.5 | 1.7 | 4.2 | 6.7 | 6.9 | 8.6 |
| | Pred. inc. | 17.1 | 57.3 | 130.0 | 220.6 | 238.3 | 119.2 |
| | $\Phi$ | 0.64 | 1.74 | 5.38 | 8.29 | 7.57 | 9.28 |
| | $n$ | 59 | 82 | 60 | 37 | 30 | 16 |
| >30 | Mean bias | –0.2 | 1.8 | –3.0 | –22.2 | –8.2 | –8.1 |
| | s.e. (bias) | 0.4 | 3.1 | 4.7 | 7.6 | 10.9 | 9.7 |
| | Pred. inc. | 14.7 | 105.5 | 204.0 | 277.6 | 236.0 | 123.5 |
| | $\Phi$ | 0.60 | 3.11 | 2.67 | 7.80 | 11.38 | 10.70 |
| | $n$ | 61 | 32 | 30 | 34 | 37 | 16 |

Overall, the bias in forecasts of volume was –1.0% (s.e. 0.4%) and the standard error of a forecast for an increment over 5 years of 119 $m^3$/ha (i.e., the average for thinned *PGP* plots) was 22.8 $m^3$/ha.

## DBH Distribution Model (Stand Table Generation)

The approach used to model the diameter distribution was a composite of the parameter recovery and parameter prediction methods described in general by Hyink & Moser (1983). The three-parameter Weibull *pdf* was adopted here to describe the DBH distribution represented by the DBH class frequencies described earlier. For the fit of the model, the observed is compared to the predicted distribution immediately prior to thinning (for an at-thinning measurement). The three-parameter Weibull *pdf* is given by

$$f(D,c,\theta,d_0) = c\theta^{-1} \{ (D-d_0)/\theta \}^{c-1} exp[- \{ (D-d_0)\theta \}^{c}].$$

where $D$ is DBHob and c, $\theta$, and $d_0$ are the shape, scale, and location parameters respectively. The corresponding cumulative density function (*cdf*) is given by

$$F(D,c,\theta,d_0) = 1 - exp[- \{ (D-d_0)/\theta \}^{c}].$$

An alternative parametrisation of the above *cdf* is given by

$$F(D,c,\theta,d_0) = 1 - exp[-exp \{ \theta^* + c\ ln(D-d_o) \} ] \qquad (14)$$

where $\theta^* = -c\ ln\theta$.

If $d_0$ is assumed known this parametrisation is a *glm* referred to as the complementary log-log link model.

The first stage in deriving the DBH distribution model was to identify a model for the shape parameter in terms of stand variables which could be forecast from the models already described. To identify such a model an estimate of the shape parameter, *c*, was obtained for each measurement within each plot to give 847 estimates of *c*. The data used in the fit were the cumulative number of stems across DBH classes. The form of the model fitted is given by Eqn (14). An assumed binomial distribution was used for the class cumulative frequency with binomial denominator given by $N^*_k$, the number of live stems on the plot. The GENSTAT programme was used to fit this model. In this fit, as well as the shape parameter, the scale parameter $\theta$ was estimated via $\theta^*$. The location parameter, $d_0$, was not estimated but was fixed by taking its value as the lower limit of the first DBH class to have a non-zero frequency. The scale and location parameters were not of interest in themselves at this stage as will be explained later. The resulting 847 estimates of *c* obtained above were then regressed on other stand variables and the best model was found to be that given by

$$c = exp(\beta_0 + \beta_1 H + \beta_2 N + \beta_3 Q) \qquad (15)$$

where $Q$ is the quadratic mean DBH calculated as $Q = \sqrt{\{B/(K\ N)\}}$ and K is the constant $\pi/[4(100^2)]$ required to convert $\text{DBH}^2$ to basal area. The subscript referring to the measurement number where absent is assumed to be $k$ for the following since the forecast of the Weibull parameters is required at age $t_k$. The parameter estimates obtained in the fit of Eqn (15) were not the final estimates but were used as initial estimates in the next stage of the estimation procedure which involved simultaneously re-estimating the parameters in Eqn (15) and recovering the scale parameter. An estimate of the location parameter, $d_0$, is also required. The procedure, adopted above, of fixing this parameter using the first non-zero DBH class obviously cannot be used at the forecast age so various models of $d_0$ were tried. It was found that the best and simplest approach was to fix $d_0$ as the lower limit of the first non-zero class at age $t_{k-1}$, which in practice means that $d_0$ is calculated from the stand table at the start of a forecast period. In the case of a forecast period beginning with a thinning, $d_0$ is obtained from the after-thinning stand table.

The final model was fitted using the generalised optimisation procedure in GENSTAT where the above model for *c* was fitted, the value of $d_0$ was obtained from the last measurement as described above, and simultaneously the value of $\theta$ was recovered using $Q$ as follows.

The value of $Q$ predicted from the Weibull, $Q_w$, is obtained using the following integration

$$Q_w^2 = \int_{d_0}^{\infty} D^2 f\,(D,c,\theta,d_0)dD$$
$$= \mu_d^2 + \sigma_d^2,$$

where $\mu_d$ and $\sigma_d^2$ are the mean and variance of the above Weibull distribution,

$$\mu_d = d_0 + \theta\,\Gamma(1 + 1/c)$$
$$\sigma_d^2 = \theta^2\{\,\Gamma(1 + 2/c) - \Gamma^2\,(1 + 1/c)\}$$

and $\Gamma(.)$ is the gamma function. Therefore the scale parameter, $\theta$, can be recovered, given estimated values of $c$ and $d_0$, by equating $Q_w$ to the observed value of $Q$ which requires solving the following quadratic for $\theta$.

$$\theta^2\Gamma(1+2/c) + 2\theta\ \Gamma(1+1/c)d_0 + d_0^2 - Q^2 = 0 \tag{16}$$

To solve the above quadratic in the optimisation procedure set up in GENSTAT the following approximation to the gamma function used by Garcia (1981) was used where

$$\begin{aligned}\Gamma(1+x) &\cong 1 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 + \alpha_5 x^5, && 0 \leq x \leq 1\\ &= x\Gamma(x) && x > 1,\end{aligned}$$

$\alpha_1 = -0.5749$, $\alpha_2 = 0.9512$, $\alpha_3 = -0.6999$, $\alpha_4 = 0.4246$ and $\alpha_5 = -0.1011$.

The function that was minimised was the deviance as defined for a binomial distribution for the observed cumulative frequencies conditional on $N_k$*. Examination of the Pearson residuals suggested the binomial variance function was more appropriate than that given by Eqn (3). The estimates of the parameters in Eqn (15) obtained in the above optimisation along with their standard errors are given in Table 8. The extra

**TABLE 8–Parameter estimates for Weibull shape parameter (c) (Eqn 15)**
$c = exp\ (\beta_0 + \beta_1 H + \beta_2 N + \beta_3 Q)$

| Parameter | Estimate | Standard error |
|---|---|---|
| $\beta_0$ | 1.2247 | 0.0393 |
| $\beta_1$ | −0.01865 | 0.00232 |
| $\beta_2$ | $-0.2168 \times 10^{-3}$ | $0.0370 \times 10^{-3}$ |
| $\beta_3$ | 0.01909 | 0.00239 |
| | $\hat{\Phi} = 1.1333$ | |

effort above to obtain estimates of these parameters while simultaneously recovering the scale parameter was worthwhile since it resulted in a 27% decrease in the residual deviance compared to that obtained using the initial estimates. However, the estimates of the standard errors of the initial estimates are given in Table 8 since it was felt that the equivalent estimates obtained in the simultaneous fit were gross under-estimates. The reason for this is probably the fact that the observed cumulative frequencies are interdependent for a plot-measurement whereas the individual plot-measurement estimates of $c$ are more independent of each other.

To examine graphically the goodness of fit of the Weibull model, the DBH class frequency data and the corresponding forecast frequencies were aggregated to give the three-way table of DBH by quadratic mean DBH by thinning classes. To do this the quadratic mean DBH, $Q$, for each measurement within each plot was grouped into 5-cm classes between 20 and 40 cm and including < 20-cm and > 40-cm classes. Within each cell of the DBH by quadratic mean DBH table the data were separated according to whether the measurement occurred before or after the first thinning. The relative observed and forecast frequencies for the above classes for measurements after thinning are shown in Fig. 4 where the observed rather than the forecast value of $B_k$ was used in the parameter recovery procedure. The corresponding results for measurements before thinning are not shown but the fit of the Weibull appeared just as adequate as that seen in Fig. 4.

Alternative methods of predicting/recovering parameters of the Weibull distribution to that used above were employed by Garcia (1984) and Knoebel *et al.* (1986 and references therein).

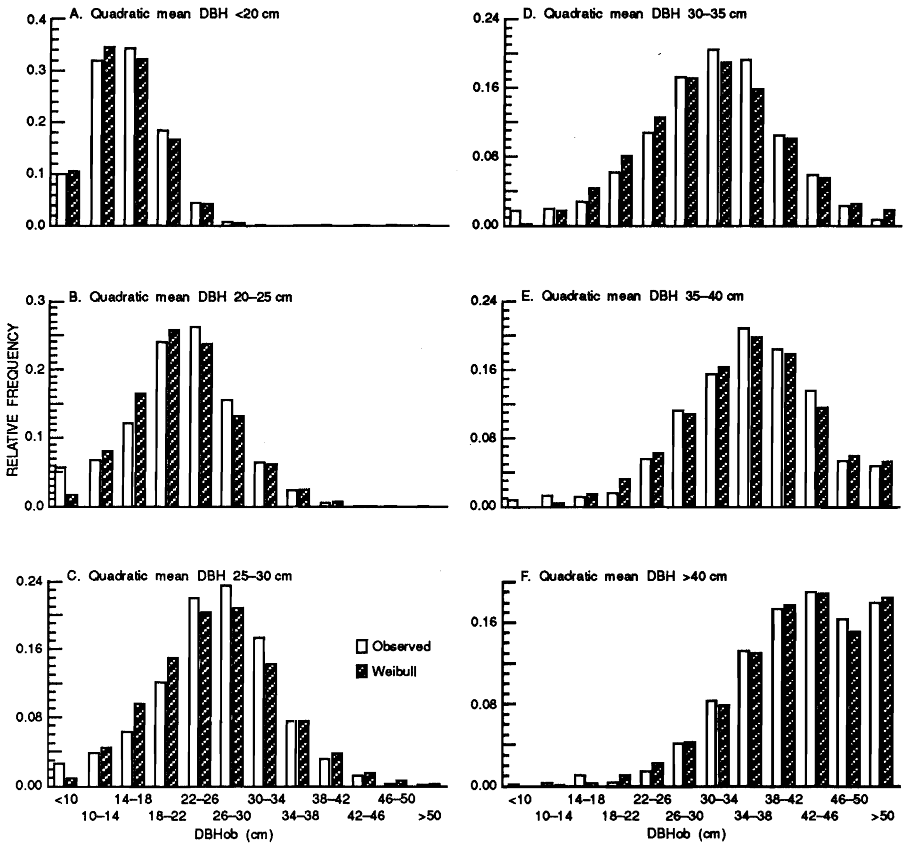


FIG. 4—Relative stocking frequency by DBH class for thinned stands.

## Thinning Model

After forecasting the stand variables up to the thinning age and determining the before-thinning DBH distribution as described above, the after-thinning stand table can be obtained by the user manually removing a number of stems from each DBH class or by allowing a thinning model (or algorithm) to be applied. In the latter the proportion, $P_n$, of the total number of stems to be removed is specified. The thinning model distributes the number of stems removed, across the DBH classes. There are two obvious constraints that need to be incorporated in this model. Firstly, the number of stems removed in a DBH class should be less than or equal to the number of stems in the class before thinning. Note that this constraint might not be satisfied if, rather than apply the thinning model below, the after-thinning stand table is obtained by applying Eqn (14), (15), and 16) using after-thinning values of *H, B,* and *N*. Secondly, the total of removals across the DBH classes should sum to the total removed which is input to the model. The model used here is given by

$$P_j = (1-P_n)exp\left\{-exp(\eta)\right\} + P_n \quad j_1 \leq j < j_2 \qquad (17)$$
$$= 0 \qquad j < j_1, j > j_2$$

where $P_j$ is the cumulative removals as a proportion of the cumulative number of stems up to and including the *j*th DBH class, $\eta$ is given by

$$\eta = \beta_1 + (\beta_2 + \beta_3 Q) ln \left\{ D_j - d_1)/(d_2 - d_1) \right\}$$

$j_1$ is the number of the first non-zero DBH class with corresponding class lower limit of $d_1$, $j_2$ is the number of the last non-zero DBH class with corresponding class upper limit $d_2$, and $D_j$ is the mid-point of the *j*th DBH class. The value of $P_{j_2}$ can be obtained by difference. The class removals can be obtained from the $P_j$'s by multiplying $P_j$ by the cumulative number of stems up to and including the *j*th class and then calculating first differences. The above model satisfies both constraints mentioned above and it was fitted in *GENSTAT* with *IRLS* based on an assumed binomial distribution for the cumulative number of stems removed. The parameter estimates are given in Table 9.

TABLE 9–Parameter estimates for thinning model (Eqn 17)

$$P_j = (1-P_n)exp\{-exp(\eta)\} + P_n \quad j_1 \leq j < j_2$$
$$= 0 \quad j < j_1, j > j_2$$

| Parameter | Estimate | Standard error |
|---|---|---|
| $\beta_1$ | 1.4322 | 0.0393 |
| $\beta_2$ | 0.7302 | 0.0729 |
| $\beta_3$ | 0.0179 | 0.0031 |
| | $\hat{\Phi} = 2.4179$ | |

## APPLICATION OF THE MODELS

To apply the above models to temporary inventory plot data so that forecasts of required forest statistics can be obtained, the following steps are required:

(1) Initial values of the following stand variables are obtained at the measurement of the inventory plot: $H, B, V, N, S, S_d, S_t, P_r$, and $d_0$. These variables are calculated after thinnings are removed if the measurement is taken at a thinning. These values are re-initialised only when a simulated treatment of the stand is applied.

(2) Using the increment models for *H* [Eqn (5)] and *B* [Eqn (9) and (10)] forecast these variables to the required age using initial values obtained in Step (1). If the

stand has not been thinned, calculate mortality using Eqn (11) otherwise assume no mortality, or apply an average mortality rate for thinned stands.

(3) Using the forecast values of $H$ and $B$ obtained in Step (2) apply the increment model for $V$ given by Eqn (12) for unthinned and Eqn (13) for thinned stands. Eqn (13) requires the initial values of $H$ and $B$ obtained in Step (1) as input as well as forecast values.

(4) Predict the parameters of the Weibull DBH distribution using Eqn (15) for $c$ using the values of $B$, $H$, and $N$ obtained in Step (2) (i.e., $Q$ obtained from $B$ and $N$), recover $\theta$ using Eqn (16) and $d_0$ obtained in Step (1). The *cdf* form of the model, Eqn (14), can then be applied with class frequencies obtained by first-differences from the forecast cumulative frequencies. For a thinning, the stand table is constructed immediately prior to thinning. A stand table for each of basal area and volume can be constructed by calculating these values at the upper and lower bounds of each DBH class (for volume use a single-tree volume equation), averaging these two boundary values for the class, and then multiplying the result by the proportion of stems in the class. Some scaling will then be necessary to ensure that the class values of $B$ and $V$ total to the values forecast above.

(5) Simulation of a thinning is carried out either manually by the user or by the thinning algorithm. Both the DBH stand table and $Q$ obtained in Step (4) as well as the proportion of total stems to be removed are the inputs. If row thinning is to be simulated, equal proportions of total removals are thinned from each DBH class. If thinning from below then the model given by Eqn (17) is used. For row thinning combined with thinning-from-below between outrows, the row thinnings are removed first and then Eqn (17) is used to thin between the outrows. Thinned basal area and volume by DBH class are obtained in the same way that before-thinning values were obtained in Step (4). The after-thinning stand table is obtained by removing thinnings from the before-thinning stand table. The standing, live, basal area and volume are re-calculated as initial values [Step (1)] for the next forecast period by accumulating the after-thinning DBH class basal areas and volumes. This allows Eqn (13) to take thinning into account, as mentioned in the section on the volume increment model. Likewise, $d_0$ is re-calculated from the after-thinning stand table for use in the next forecast period.

(6) The plot is "grown on" to the next age by repeating Steps (1) to (5).

## DISCUSSION AND CONCLUSIONS

The Poisson-like variance function ($\lambda = 1$) was chosen for the MDH, basal area, and volume increment models; however, the choice between $\lambda = 1$ and $\lambda = 2$ is not clear-cut. The standard deviations of Pearson residuals, standardised to have unit variance, for classes of predicted increment, are shown in Fig. 5. These standard deviations have been averaged across the three increment models. To present them on the same predicted increment scale, the scale of the abscissa in Fig. 5 represents the ratio of the mean predicted increment for the class to the mean for the top class. These ratios, like the standard deviations, were averaged across the three models. This was done for brevity and because the trends in standard deviations were very similar for each model.

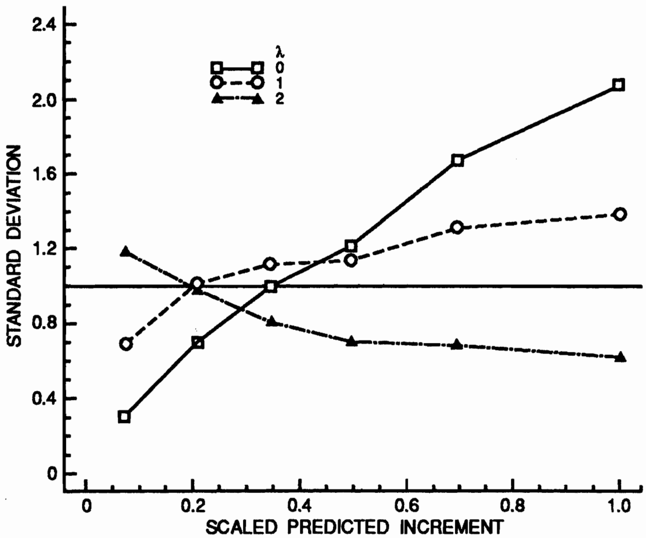


**FIG. 5—Standard deviation of Pearson residuals for predicted increment classes.**

Setting $\lambda = 1$ under-corrects for the variance trend of the standard residuals ($\lambda = 0$) while $\lambda = 2$ over-corrects (Fig. 5). This partly explains the trend of increasing $\Phi$ with age and site index seen in Table 7, since a similar trend in bias is also responsible for this trend in $\Phi$ . The value $\lambda = 1$ was chosen here because it is conservative, lying between the extremes of 0 and 2. The value $\lambda = 2$ is a more convenient form of the variance function since the standard error of forecasts can be quoted as a proportion (i.e., $\sqrt{\hat{\Phi}}$) of predicted increment. It appears that a value of $\lambda$ between 1 and 2 would be more appropriate. Further refining of the statistical methodology presented here could make use of recent work in extending the definition of the quasi-likelihood function (Nelder & Pregibon 1987) which allows $\lambda$ to be estimated along with other model parameters.

The volume increment function, Eqn (13), can be expressed in terms of ratios. Since $\hat{\beta}_2$ is effectively 1.0, the model predicts the ratio of volumes at two ages as the product of the corresponding ratios for each of basal area and the power $\hat{\beta}_1$ of mean dominant height. The relative merits of the two forms of volume model require further investigation.

A set of growth and yield models, involving increment models of mean dominant height, basal area, mortality, and volume and models of DBH distribution and thinning, have been constructed for application to intensively managed *P. radiata* plantations in Tasmania. These models, along with existing models of tree volume, stem taper, bark thickness, maximum basal area, and the tree total height/DBH relationship, allow the simulation of stands treated by thinning or pruning for any site and/or regime in Tasmania. The models are designed to be applied using temporary inventory plot data as the starting point in simulations. Because of the generality of the models the output they produce in simulations can be considered only as base-line information so that extra information on regional, fertiliser, and irregular mortality effects may be required to improve the precision of forecasts.

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## APPENDIX

The generalisation of the residual sum of squares for normally distributed, constant error variance models to the deviance for the exponential family of distributions was derived by Nelder & Wedderburn (1972). Subsequently, Wedderburn (1974) introduced the quasi-deviance statistic (the term "deviance" is used for brevity) as a further generalisation of likelihood to quasi-likelihood where the distributional form of the data is unspecified and a mean/variance relationship, such as that given by Eqn (2) is all that is assumed. The deviance (or quasi-deviance) is derived from Wedderburn's (1974) quasi-likelihood as

$$D(y,\hat{\mu}_k) = -2\,\Sigma^{n}_{k=1} \int_{y_k}^{\hat{\mu}_k} [(y_k-\mu)/V(\mu)]\ d\mu$$

where $y_k$ is the dependent variable, $\hat{\mu}_k$ is the predicted value from the model $\mu$, V(.) is the variance function, $n$ is the number of observations, and $D(y,\hat{\mu}_k)$ is the deviance for the model under consideration.

The deviance function for the gamma-like variance function, given by Eqn (2 with $\lambda = 2$, for a single observed/forecast pair is given by

$$D(y_k,\hat{\mu}_k) = 2\ [(y_k-\hat{\mu}_k)/(\hat{\mu}_k-y_{k-1}) - ln\ \{(y_k-y_{k-1})/(\hat{\mu}_k-y_{k-1})\}\ ]\quad ,\ y_k > y_{k-1}$$

Similarly, the deviance function for the Poisson-like variance function, given by Eqn (2) with $\lambda = 1$, is given by

$$D(y_k,\hat{\mu}_k) = 2\ [(y_k-y_{k-1})ln\ \{(y_k-y_{k-1})/(\hat{\mu}_k-y_{k-1})\} - (y_k-\hat{\mu}_k)]\qquad ,\ y_k > y_{k-1}$$

The estimate of the dispersion parameter, $\Phi$, used here was the mean deviance, so that $\hat{\Phi} = \Sigma_k D(y_k,\hat{\mu}_k)/n$. An approximate $100(1-\alpha)\%$ confidence interval for $\hat{\Phi}$ is

$$(n-p)\hat{\Phi}/\chi^2_{n-p}(1-\alpha) < \Phi < (n-p)\hat{\Phi}/\chi^2_{n-p}(\alpha)$$

where $\chi^2_{n-p}(1-\alpha)$ is the $100(1-\alpha)$ percentage point of the chi square distribution with $(n-p)$ degrees of freedom and $p$ is the number of parameters estimated.